

# Grade 4 - Module 3

## Multi-Digit Multiplication and Division

Topic A: Multiplicative Comparison Word Problems:

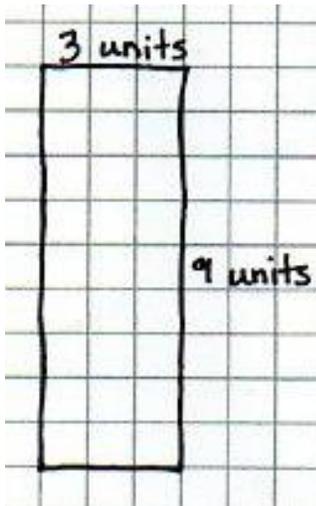
**Perimeter** – length of a continuous line forming the boundary of a closed geometric figure

Formulas for  
Perimeter

$$P = L + w + L + w$$

$$P = 2L + 2w$$

$$P = 2 \times (L + w)$$

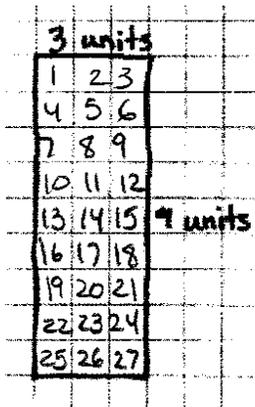


-Add up the length of all of the sides.  $3 + 9 + 3 + 9 = 24$ . The perimeter is 24 units. → You could also add  $3 + 3 + 9 + 9$ . The answer is still 24 units. The order doesn't matter when you are adding.

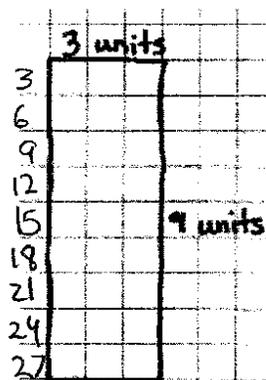
-We can also add the length and width first, then multiply that sum by 2. What is the length plus width of this rectangle? 3 plus 9 equals 12. 12 units. 12 units doubled, or 12 units times 2, equals? 24 units.

## Multiplicative Comparison Word Problems (continued):

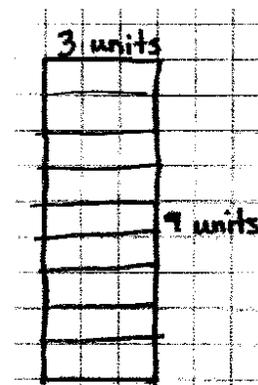
**Area** – the amount of two-dimensional space in a bounded region



27 square units



27 square units



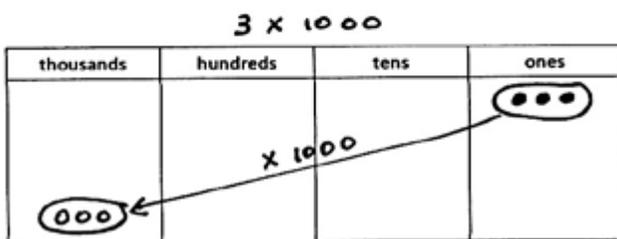
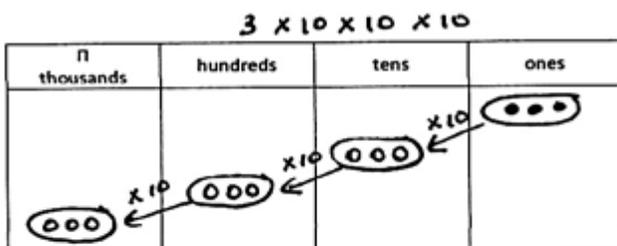
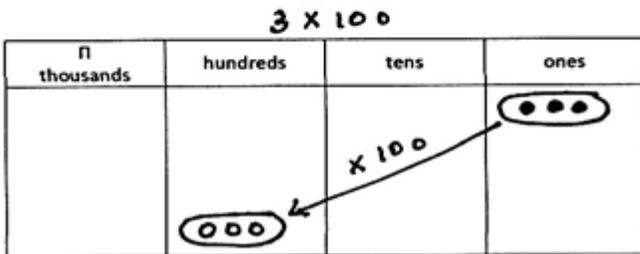
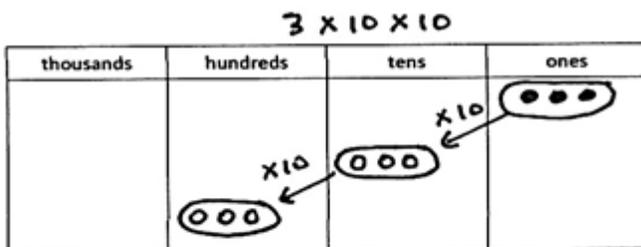
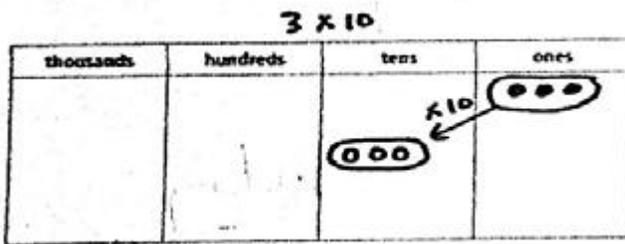
$$9 \times 3 = 27$$

27 square units

We can count all of the squares. → We could also count the number of squares in one row and then skip count that number for all of the rows. → That's just multiplying the number of rows by the number in each row. → A quicker way is to multiply the length times the width. Nine rows of 3 units each is like an array. We can just multiply  $9 \times 3$ .

$$\text{Area} = l \times w$$

## Topic B: Multiplication by 10, 100, and 1,000:



- Suppose I wanted to multiply 3 ones by ten. We can just move each disk over to the tens place and get 3 tens.

- What if I wanted to multiply that by 10? Move them one more place into the hundreds and get 3 hundreds.

- I started with 3 ones. What did I multiply 3 ones by to get 3 hundreds? That's just  $3 \times 100$ .

- I showed 3 times 1,000 by showing  $3 \text{ ones} \times 10$  to get 3 tens. Then I did times 10 again to get 3 hundreds and times 10 again to show 3 thousands.  $\rightarrow$  I drew an arrow representing *times 1,000* from 3 ones to the thousands column.

$$10 \times 20 \times 3$$

hundreds	tens	ones

$$10 \times 30 \times 2$$

hundreds	tens	ones

$$2 \times 30 \times 10$$

hundreds	tens	ones

$$3 \times 20 \times 10$$

hundreds	tens	ones

$$30 \times 20$$

- Here we are multiplying a two-digit number by another two-digit number. What are some other ways we could express  $30 \times 20$ ?

$$3 \text{ tens} \times 2 \text{ tens.}$$

$$10 \times 20 \times 3.$$

$$10 \times 30 \times 2.$$

$$2 \times 30 \times 10.$$

$$3 \times 20 \times 10.$$

- What is 2 tens times 10?

- 2 tens times 10 is 2 hundreds.

- So the value of  $10 \times 20$  is?

- 200.

- And then  $200 \times 3$ ?

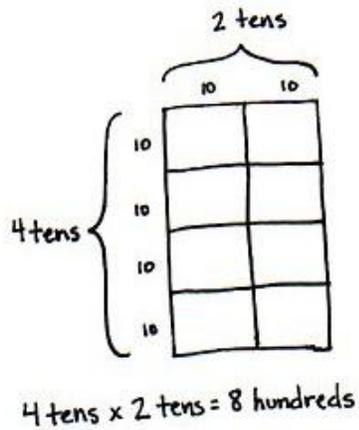
- Triple that group.  $\rightarrow$  200 times 3.  $\rightarrow$  3 times 2 hundreds.  $\rightarrow$  3 groups of 2 hundred.

-  $10 \times 20 \times 3$  is?

600.

- When we multiply a two-digit number by another two-digit number, there are many equivalent ways to express it as a product.

Decomposing our multiplication problem into more units can help us solve.



**Area Model- 40 x 20**

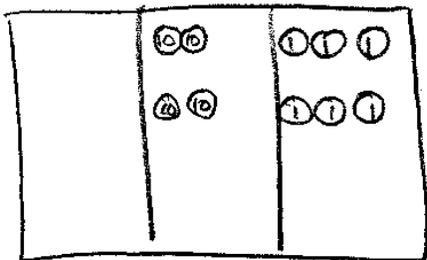
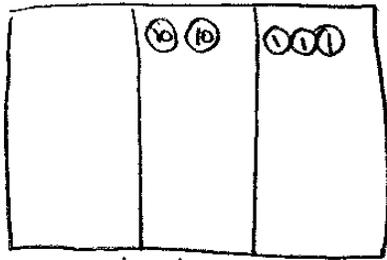
-What is 4 tens times 2 tens?

-I know 4 times 2 is 8. I don't know what to do with the units.

→ I know 4 times 2 is 8. That leaves both tens. 10 tens. It's like saying 4 times 2 times 10 tens!

-2 times 4 equals 8. Each square

**Multiplication of up to Four Digits by Single-Digit Numbers:**

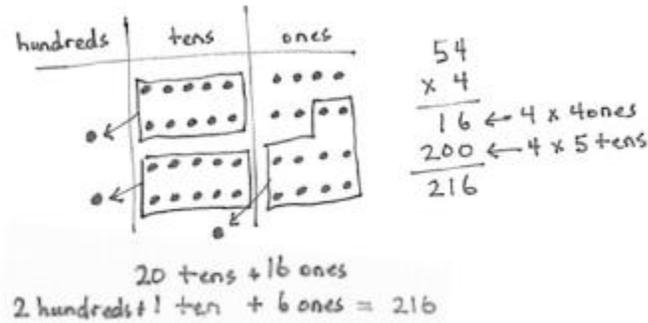


$2 \times 2 \text{ tens} + 2 \times 3 \text{ ones}$   
 $\downarrow \quad \quad \downarrow$   
 $4 \text{ tens} + 6 \text{ ones} = 46$

$$\begin{array}{r}
 23 \\
 \times 2 \\
 \hline
 6 \leftarrow 2 \times 3 \text{ ones} \\
 + 40 \leftarrow 2 \times 2 \text{ tens} \\
 \hline
 46
 \end{array}$$

## Topic C: Multiplication of up to Four Digits by Single-Digit Numbers

Use place value disks to represent multiplication: Represent  $4 \times 54$  with disks, writing a matching equation and recording the partial products vertically



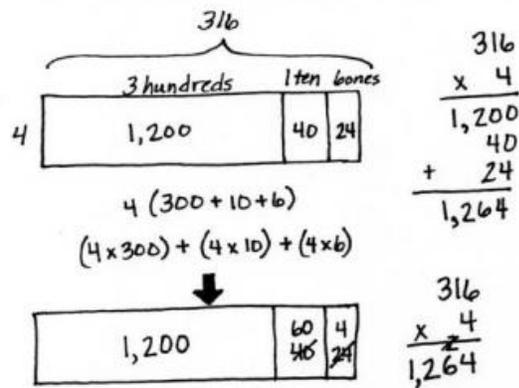
Move to abstract level using partial products, area model and standard algorithm

Solve  $5 \times 237$  using the partial products algorithm.

$$\begin{array}{r} 237 \\ \times 5 \\ \hline 35 \\ 150 \\ + 1000 \\ \hline 1,185 \end{array}$$

### Area Model

A rectangle is partitioned into hundreds, tens, and ones. I'll multiply 4 times 3 hundreds, 4 times 1 ten, and 4 times 6 ones and add the three products together for the answer. That's like the break apart and distribute property, which allows us to break apart the large multiplication problem into three smaller ones.



## Topic E: Division of Tens and Ones with Successive Remainders

Divide a two-digit number by a one-digit number modeled with an array.



There are 3 students  
on each team.

$$12 \div 4 = 3$$

$$4 \times 3 = 12$$

-or-

There are 4 teams  
of 3 students.

$$12 \div 3 = 4$$

$$4 \times 3 = 12$$

With a remainder



$$13 \div 4$$

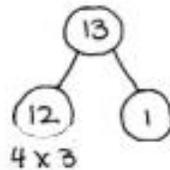
There are 3 in each group  
with 1 remaining.

The quotient is 3.  
The remainder is 1.

$$13 \div 3$$

There are 4 groups  
with 1 remaining.

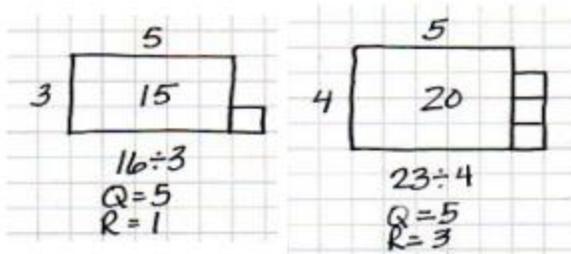
The quotient is 4.  
The remainder is 1.



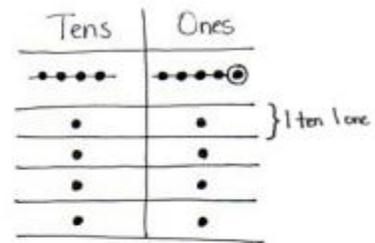
$$4 \times 3 = 12$$

$$12 + 1 = 13$$

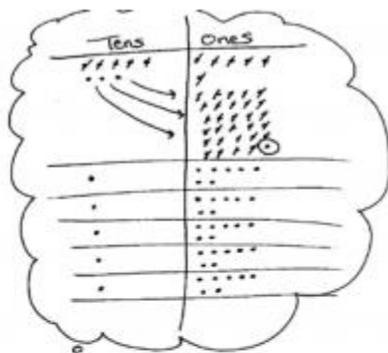
## Division using an area model:



Understand and solve two-digit dividend division problems with a remainder in the ones place by using number disks.



$$\begin{array}{r}
 11 \text{ R}1 \\
 4 \overline{)45} \\
 \underline{-4} \phantom{0} \\
 05 \\
 \underline{-4} \\
 1
 \end{array}$$



Divide with a remainder in the tens and ones place.

$$\begin{array}{r}
 17 \text{ R}1 \\
 5 \overline{)86} \\
 \underline{-5} \phantom{0} \\
 36 \\
 \underline{-35} \\
 1
 \end{array}$$

Decompose  $48 \div 4$  from whole to part.

Draw a rectangle with an area of 48 square units and a width of 4 units. Then draw a number bond to match the whole and parts of the rectangle.

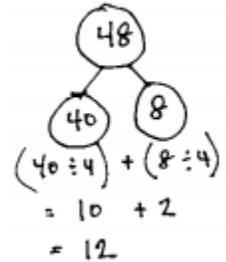
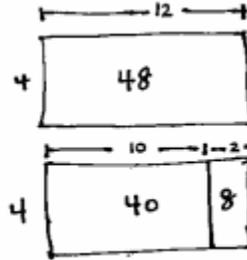
$$40 \div 4 = 10$$

$$8 \div 4 = 2$$

$$10 + 2 = 12$$

What is 48 divided by 4?

12.



2  $2 \overline{)37}$  *2 x \_\_\_ tens = 3 tens*

2  $2 \overline{)37}$   $\begin{array}{r} 1 \\ -2 \\ \hline 1 \end{array}$

2  $2 \overline{)37}$   $\begin{array}{r} 18 \\ -2 \\ \hline 17 \\ -16 \\ \hline 1 \end{array}$  *1 ten + 7 ones = 17 ones*  
*2 x \_\_\_ ones = 17 ones*

2  $2 \overline{)37}$   $\begin{array}{r} 18 \\ -2 \\ \hline 17 \\ -16 \\ \hline 1 \end{array}$  *One square unit remains. It doesn't make another whole side length.*  
*1 square unit remaining*

The length of the unknown side is 18 units.  
One square unit was leftover.

$$\begin{aligned} (20 \div 2) + (16 \div 2) \\ = 10 + 8 \\ = 18 \end{aligned}$$

$$(18 \times 2) + 1 = 37$$

## Topic F: Reasoning with Divisibility

- Composite number (positive integer having three or more whole number factors) 4, 6, 8, 10, etc.
- Prime number (positive integer only having whole number factors of one and itself) 2, 3, 5, 7, etc.

## Topic G: Division of Thousands, Hundreds, Tens, and Ones

$$\begin{array}{l} \overline{)1344} \\ \underline{6} \\ 1200 \\ \underline{6} \\ 120 \\ \underline{6} \\ 24 \\ \underline{6} \\ 0 \end{array}$$

$$\begin{array}{l} 1344 \\ \swarrow \quad \downarrow \quad \searrow \\ 1200 \quad 120 \quad 24 \\ \hline (1200 \div 6) + (120 \div 6) + (24 \div 6) \\ = 200 + 20 + 4 \\ = 224 \end{array}$$

$$\begin{array}{l} \overline{)1344} \\ \underline{6} \\ 600 \quad 600 \quad 60 \quad 24 \\ \underline{6} \\ 0 \end{array}$$

$$\begin{array}{l} 1344 \\ \swarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \searrow \\ 600 \quad 600 \quad 60 \quad 60 \quad 24 \\ \hline (600 \div 6) + (600 \div 6) + (60 \div 6) + (60 \div 6) + (24 \div 6) \\ = 100 + 100 + 10 + 10 + 4 \\ = 224 \end{array}$$

## Topic H: Multiplication of Two-Digit by Two-Digit Numbers

$$\begin{array}{r} 20 \quad 6 \\ 4 \quad 4 \times 20 \quad 4 \times 6 \\ 30 \quad 30 \times 20 \quad 30 \times 6 \\ \hline \end{array}$$

$$\begin{array}{l} 4 \text{ partial products} \\ (4 \times 6) + (4 \times 20) + (30 \times 6) + (30 \times 20) \\ = 24 + 80 + 180 + 600 \\ = 104 + 780 \\ = 884 \end{array}$$

$$\begin{array}{r} 26 \\ 4 \quad 4 \times 26 \\ 30 \quad 30 \times 26 \\ \hline \end{array}$$

$$\begin{array}{l} 2 \text{ partial products} \\ (4 \times 26) + (30 \times 26) \\ = 104 + 780 \\ = 884 \end{array}$$

$$\begin{array}{r} 26 \\ \times 34 \\ \hline 24 \\ 80 \\ 180 \\ + 600 \\ \hline 884 \end{array}$$

$$\begin{array}{r} 26 \\ \times 34 \\ \hline 104 \\ + 780 \\ \hline 884 \end{array}$$